Fisher's LSD (Least Significant Difference)

Fisher's LSD is a method for comparing treatment group means after The ANOVA null hypothesis of equal means has been rejected using the ANOVA F-test. If the F-test fails to reject the null hypothesis this procedure should not be used.

Anova F-test

Let's assume we have 4 treatment groups A, B, C, and D. The summary statistics for the groups are

Group	$\bar{y}$	s	n
A	74.3	5.6	10
B	82.8	5.1	10
C	77.8	5.3	10
D	82.9	4.6	10

The ANOVA table for this data is

Source	DF	SS	MS	F	p-value
Between groups	3	523.70	174.57	6.55	0.0012
Within groups (Error)	36	959.58	26.65		
Total	39	1483.28			

Since the F-test p-value is less than 0.05, we reject the null hypothesis $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$ at the 0.05 level.

Pairwise comparison of the means

At this point we are interested in doing pairwise comparisons of the means. That is, we want to test hypotheses of the sort $H_0 : \mu_A = \mu_B$, $H_0 : \mu_A = \mu_C$, etc. The LSD method for testing the hypothesis $H_0 : \mu_A = \mu_B$ proceeds as follows:

1. Calculate $LSD_{A,B} = t_{0.05/2,DFW}\sqrt{MSW(1/n_A + 1/n_B)}$ Where DFW = DF(within) and MSW = MS(within). For our example $LSD_{A,B} = 2.021\sqrt{26.65(2/10)} = 4.67$.

2. If $|\bar{y}_A - \bar{y}_B| \geq LSD_{A,B}$ then we reject the null hypothesis $H_0 : \mu_A = \mu_B$. For our example $|\bar{y}_A - \bar{y}_B| = 8.5$ which is greater than 4.67 so we reject $H_0 : \mu_A = \mu_B$ at the 0.05 level.

We then continue to test all pairs of interest. In this case the LSD is the same for all pairs because $n_A = n_B = n_C = n_D$. Thus $LSD_{A,B} = LSD_{A,C} = ... = LSD_{C,D} = 4.67$ and we compare all pairwise differences in the means to 4.67. Here are the absolute pairwise differences and the results of Fisher's LSD:

$|\bar{y}_A - \bar{y}_B| = 8.5 \geq 4.67$ reject $H_0 : \mu_A = \mu_B$
$|\bar{y}_A - \bar{y}_C| = 3.5 < 4.67$ do not reject $H_0 : \mu_A = \mu_C$
$|\bar{y}_A - \bar{y}_D| = 8.6 \geq 4.67$ reject $H_0 : \mu_A = \mu_D$
$|\bar{y}_B - \bar{y}_C| = 5.0 \geq 4.67$ reject $H_0 : \mu_B = \mu_C$
$|\bar{y}_B - \bar{y}_D| = 0.1 < 4.67$ do not reject $H_0 : \mu_B = \mu_D$
$|\bar{y}_C - \bar{y}_D| = 5.1 \geq 4.67$ reject $H_0 : \mu_C = \mu_D$